

Solving Trigonometric Equations

Solve each equation for all values of x .

1. $5\sin x + 2 = \sin x$ 2. $5 = \sec^2 x + 3$ 3. $2 = 4\cos^2 x + 1$ 4. $4\tan x - 7 = 3\tan x - 6$
 5. $9 + \cot^2 x = 12$ 6. $2 - \sec x = 4 - 9\sec x$ 7. $3\csc x = 2\csc x + \sqrt{2}$ 8. $11 = 3\csc^2 x + 7$
 9. $6\tan^2 x - 2 = 4$ 10. $9 + \sin^2 x = 10$ 11. $7\cot x - \sqrt{3} = 4\cot x$ 12. $7\cos x = 5\cos x + \sqrt{3}$

Find all solutions of each equation on $[0, 2\pi)$

13. $\sin^4 x + 2\sin^2 x - 3 = 0$ 14. $-2\sin x = -\sin x \cos x$ 15. $4\cot x = \cot x \sin^2 x$ 16. $\csc^2 x - \csc x + 9 = 11$
 17. $\cos^3 x + \cos^2 x - \cos x = 1$ 18. $2\sin^2 x = \sin x + 1$ 19. $1 = \cot^2 x + \csc x$ 20. $\sec x = \tan x + 1$
 21. $\tan^2 x = 1 - \sec x$ 22. $\csc x + \cot x = 1$ 23. $2 - 2\cos^2 x = \sin x + 1$ 24. $\cos x - 4 = \sin x - 4$
 25. $3\sin x = 3 - \cos x$ 26. $\cot^2 x \csc^2 x - \cot^2 x = 9$ 27. $\sec^2 x - 1 + \tan x - \sqrt{3}\tan x = \sqrt{3}$
 28. $\sec^2 x \tan^2 x + 3\sec^2 x - 2\tan^2 x = 3$ 29. $\frac{\tan^2 x}{\sec x} + \cos x = 2$ 30. $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = -4$
 31. $\frac{\sin x + \cos x}{\tan x} + \frac{1 - \sin x}{\sin x} = \cos x$ 32. $\cot x \cos x + 1 = \frac{1}{\sec x - 1} + \frac{\sin x}{\tan^2 x}$

Find all solutions of each equation on $[0, 4\pi)$

33. $4\tan x = 2\sec^2 x$ 34. $2\sin^2 x + 1 = -3\sin x$ 35. $\csc x \cot^2 x = \csc x$ 36. $\sec x + 5 = 2\sec x + 3$
 37. **ERROR ANALYSIS** Vijay and Alicia are solving $\tan^2 x - \tan x + \sqrt{3} = \sqrt{3}\tan x$. Vijay thinks that the solutions are $x = \frac{\pi}{4} + n\pi$, $x = \frac{5\pi}{4} + n\pi$, $x = \frac{\pi}{3} + n\pi$, and $x = \frac{4\pi}{3} + n\pi$. Alicia thinks that the solutions are $x = \frac{\pi}{4} + n\pi$, and $x = \frac{\pi}{3} + n\pi$. Is either of them correct? Explain your reasoning.

38. **REASONING** Are the solutions of $\csc x = \sqrt{2}$ and $\cot^2 x + 1 = 2$ equivalent? If so, verify your answer algebraically. If not, explain your reasoning.