

## Polynomials Finding Zeros and Factoring

Find the following for each polynomial: describe the end behavior, then find the possible real zeros, the possible imaginary zeros, the possible rational roots, the possible positive real zeros, the possible negative real zeros, the zeros and write the polynomial as a product of linear factors. Then graph.

1  $x^3 + 3x^2 - x - 3$

2  $x^3 + 5x^2 - 12x - 36$

3  $x^3 - 7x - 6$

4  $x^4 - 9x^2 + 4x + 12$

5  $x^4 - 2x^2 + 1$

6  $x^4 - x^3 - 13x^2 + x + 12$

7  $x^4 + x^3 - 10x^2 - 16x + 32$

8  $x^3 + 4x^2 - 11x - 30$

9  $x^3 + 6x^2 + 11x + 6$

10  $x^3 - x^2 - 24x - 36$

11  $x^4 - x^3 - 9x^2 - 11x - 4$

12  $x^4 - 13x^2 + 36$

13  $x^3 + 2x^2 - 16x - 32$

14  $x^4 + x^3 - 13x^2 - x + 12$

15  $2x^3 - x^2 - 11x + 6$

16  $3x^3 + 13x^2 - 11x - 5$

17  $2x^3 + 9x^2 + 3x - 4$

18  $x^3 + 6x^2 - 13x - 42$

19  $x^4 + 4x^3 - 6x^2 - 4x + 5$

20  $x^4 + 5x^3 - 7x^2 - 5x + 6$

21  $x^4 - 5x^2 + 4$

22  $2x^3 - 7x^2 - 60x + 32$

23  $x^4 + 2x^3 - 5x^2 - 6x - 72$

24  $3x^3 + 4x^2 - 5x - 2$

25  $6x^3 + 7x^2 - 1$

26  $x^4 + 10x^3 + 35x^2 + 50x + 24$

27  $x^4 + 3x^3 - 15x^2 - 19x + 30$

28  $x^4 - 2x^3 + 2x - 1$

29  $x^4 + 8x^3 + 23x^2 + 28x + 12$

30  $x^4 - 17x^2 + 16$

## Answers

- possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2; possible rational roots:  $\pm 1, \pm 3$
- 1 possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-3, -1, 1$ ;  
product of linear factors:  $(x+1)(x-1)(x+3)$
- possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;
- 2 possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$   
possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-6, 2, 3$ ;  
product of linear factors:  $(x+6)(x-3)(x-2)$
- possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2; possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 6$
- 3 possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-2, -1, 3$ ;  
product of linear factors:  $(x+1)(x-3)(x+2)$
- possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;
- possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
- 4 possible positive real zeros: 2 or 0,  
possible negative real zeros: 2 or 0; zeros:  $-3, -1, 2$ ;  
product of linear factors:  $(x+1)(x+3)(x-2)(x-2)$
- possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;
- possible rational roots:  $\pm 1$
- 5 possible positive real zeros: 2 or 0,  
possible negative real zeros: 2 or 0; zeros:  $-1, 1$   
product of linear factors:  $(x+1)(x+1)(x-1)(x-1)$
- possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;
- possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
- 6 possible positive real zeros: 2 or 0,  
possible negative real zeros: 2 or 0; zeros:  $-3, -1, 1, 4$   
product of linear factors:  $(x+3)(x+1)(x-1)(x-4)$
- possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;
- possible rational roots:  $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$
- 7 possible positive real zeros: 2 or 0,  
possible negative real zeros: 2 or 0; zeros:  $-4, -2, 1, 4$   
product of linear factors:  $(x+2)(x+4)(x-1)(x-4)$

- possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;  
possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$   
8 possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-5, -2, 3$ ;  
product of linear factors:  $(x+5)(x-3)(x+2)$
- possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;  
possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 6$   
9 possible positive real zeros: 0, possible negative real zeros: 3 or 1; zeros:  $-3, -2, -1$ ;  
product of linear factors:  $(x+1)(x+3)(x+2)$
- possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;  
possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$   
10 possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-3, -2, 6$ ;  
product of linear factors:  $(x-6)(x+3)(x+2)$
- possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;  
possible rational roots:  $\pm 1, \pm 2, \pm 4$   
11 possible positive real zeros: 1  
possible negative real zeros: 3 or 1; zeros:  $-1, 4$   
product of linear factors:  $(x+1)(x+1)(x+1)(x-4)$
- possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;  
possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm 18, \pm 36$   
12 possible positive real zeros: 2 or 0  
possible negative real zeros: 2 or 0; zeros:  $-3, -2, 2, 3$   
product of linear factors:  $(x+3)(x+2)(x-3)(x-2)$
- possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;  
possible rational roots:  $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$   
13 possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-4, -2, 4$ ;  
product of linear factors:  $(x+4)(x-4)(x+2)$
- possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;  
possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$   
14 possible positive real zeros: 2 or 0,  
possible negative real zeros: 2 or 0; zeros:  $-4, -1, 1, 3$   
product of linear factors:  $(x-3)(x+1)(x-1)(x+4)$

possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;

possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

15

possible positive real zeros: 0, possible negative real zeros: 3 or 1; zeros:  $-2, \frac{1}{2}, 3$ ;

product of linear factors:  $(x+2)(x-3)(2x-1)$

possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;

possible rational roots:  $\pm 1, \pm 5, \pm \frac{1}{3}, \pm \frac{5}{3}$

16

possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-5, -1, -\frac{1}{3}$ ;

product of linear factors:  $(x+5)(x-1)(3x+1)$

possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;

possible rational roots:  $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$

17

possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-4, -1, \frac{1}{2}$ ;

product of linear factors:  $(x+1)(x+4)(2x-1)$

possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;

possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42$

18

possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-7, -2, 3$ ;

product of linear factors:  $(x+7)(x-3)(x+2)$

possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;

possible rational roots:  $\pm 1, \pm 5$

19

possible positive real zeros: 2 or 0,

possible negative real zeros: 2 or 0; zeros:  $-5, -1, 1$ ;

product of linear factors:  $(x+5)(x-1)(x-1)(x+1)$

possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;

possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 6$

20

possible positive real zeros: 2 or 0,

possible negative real zeros: 2 or 0; zeros:  $-6, -1, 1$ ;

product of linear factors:  $(x+1)(x+6)(x-1)(x-1)$

possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;

possible rational roots:  $\pm 1, \pm 2, \pm 4$

21 possible positive real zeros: 2 or 0

possible negative real zeros: 2 or 0; zeros:  $-2, -1, 1, 2$

product of linear factors:  $(x+2)(x-2)(x-1)(x+1)$

possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;

possible rational roots:  $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$

22 possible positive real zeros: 2 or 0, possible negative real zeros: 1; zeros:  $-4, \frac{1}{2}, 8$ ;

product of linear factors:  $(x-8)(x+4)(2x-1)$

possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;

possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 18, \pm 24, \pm 36, \pm 72$

23 possible positive real zeros: 1

possible negative real zeros: 3 or 1; zeros:  $-4, -3, 2, 3$

product of linear factors:  $(x+3)(x+4)(x-3)(x-2)$

possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;

possible rational roots:  $\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

24 possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-2, 1, -\frac{1}{3}$ ;

product of linear factors:  $(x+5)(x-1)(3x+1)$

possible real zeros: 3 or 1; possible imaginary zeros: 0 or 2;

possible rational roots:  $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$

25 possible positive real zeros: 1, possible negative real zeros: 2 or 0; zeros:  $-1, -\frac{1}{2}, \frac{1}{3}$ ;

product of linear factors:  $(2x+1)(x+1)(3x-1)$

possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;

possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

26 possible positive real zeros: 0

possible negative real zeros: 4, 2 or 0; zeros:  $-4, -3, -2, -1$

product of linear factors:  $(x+2)(x+4)(x+1)(x+3)$

possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;

possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

27 possible positive real zeros: 2 or 0,

possible negative real zeros: 2 or 0; zeros:  $-5, -2, 1, 3$

product of linear factors:  $(x-3)(x+2)(x-1)(x+5)$

possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;

possible rational roots:  $\pm 1$

28 possible positive real zeros: 3 or 1,

possible negative real zeros: 1; zeros:  $-1, 1$

product of linear factors:  $(x-1)(x+1)(x-1)(x-1)$

possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;

possible rational roots:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

29 possible positive real zeros: 0,

possible negative real zeros: 4, 2 or 0; zeros:  $-3, -2, -1$ ;

product of linear factors:  $(x+2)(x+2)(x+1)(x+3)$

possible real zeros: 4, 2 or 0; possible imaginary zeros: 0, 2 or 4;

possible rational roots:  $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

30 possible positive real zeros: 2 or 0

possible negative real zeros: 2 or 0; zeros:  $-4, -1, 1, 4$ ;

product of linear factors:  $(x+4)(x+1)(x-4)(x-1)$